If both classes are supposed finite, we may ask what is the probable rank, K, in a class of N students of the kth student in a class of n students. We have $K/(N+1) = k/(n+1)$, or $K = k(N+1)/(n+1)$. Thus, for example, the second student in a class of four, according to the above assumptions would stand at a distance of 2/5 from the origin, and would rank the same as the eighteenth student in a class of 44.

BOOK REVIEWS.

EDITED BY W. H. BUSSEY, University of Minnesota.

The Development of the Arabic Numerals in Europe Exhibited in Sixty-four Tables. By G. F. HILL. Oxford, Clarendon Press, 1915.

In *Archæologia*, Vol. LXII, Mr. G. F. Hill, keeper of the department of medals and coins in the British Museum, published a series of fifty-one tables of Hindu-Arabic numerals as they appeared in Mss. and on monuments, coins, seals, medals, brasses, and paintings, as well as a few forms from printed works. This article has been of great value to all who are interested in the exact dating of mediæval manuscripts, for the work furnishes, where numerals are used, definite checks upon the time when a manuscript was written. Further and more particularly, the work is of interest to students of the history of science for it shows in graphical form the slow but sure progress of the Hindu arithmetic through Europe. Hearty welcome, then, is given to the present publication of the Oxford Press, in which appear not only the tables of the earlier work, but also additional tables of forms more recently located.

In general the forms included are only those which antedate 1500 A.D., for by that time the new arithmetic had become well-nigh universal. Parenthetically it is of interest to note, however, that as late as 1540, Köbel, in Germany, published an arithmetic wholly, except paging, in Roman numerals.

The earliest European forms are doubtless those found in the *Codex Vigilanus*, written 976 A.D. in the monastery of Albelda near Logrono in Spain. A second Spanish manuscript of about the same date, not described by Mr. Hill, also contains similar forms, and facsimiles. Both are to appear in the next issue of Professor John M. Burnam's *Palæographia iberica*. The numerals appear to have been known in Syria in 662 A.D., for a Syrian Bishop, Severus Sebokt, of a monastery on the Euphrates, refers in a work of that time to the "easy method of their, *i. e.*, the Hindus', calculations and of their computations which surpasses words. I mean that made with nine symbols," referring certainly to the Hindu system of numerals with the zero.

Interesting also is the fact that Mr. Hill finds that the forms which afford the best criteria for dating are those for 2, 4, and 7, while 5, "the most freakish of all figures," comes next. In probably the earliest translation of an Arabic treatise on the Hindu arithmetic, the *Algoritmi de numero Indorum* (published by Bon-

compagni in 1857), a translation made quite certainly in the early part of the twelfth century, the unknown writer states that there is some diversity among men in writing the figures for 5, 6, 7, and 8. This may even have been in Al-Khowarizmi's lost original treatise on arithmetic; no forms are found at this point in the unique Cambridge MS. of the translation. In passing we may note that the 2 written in form like a 7 is indicative of writing earlier than about 1,350 while a three which resembles a printed *r* (small *r*) is characteristic of the twelfth or early thirteenth century.

Mr. Hill has previously established a reputation in the field of numismatics, particularly Greek, and in Greek history. In the present treatise Mr. Hill has rendered a real service to the history of science by his patient and long-continued search for early appearances of the Hindu-Arabic numerals in Europe and by their publication in this convenient form.

The price of the book is $1.00; and it may be obtained from the Oxford University Press, 29 W. 32d St., New York City.

LOUIS C. KARPINSKI.

PROBLEMS AND SOLUTIONS.

EDITED BY B. F. FINKEL AND R. P. BAKER.

PROBLEMS FOR SOLUTION.

ALGEBRA.

445. Proposed by S. A. JOFFE, New York City.

Sum the series

$$\binom{cn}{a} - \binom{a}{1}\binom{cn-c}{a} + \binom{a}{2}\binom{cn-2c}{a} - \cdots + (-1)^a\binom{cn-ca}{a}.$$

446. Proposed by CLIFFORD N. MILLS, Brookings, South Dakota.

Solve the equations

$$x^2(y-z) = l^2(m-n), \qquad y^2(z-x) = m^2(n-l), \qquad z^2(x-y) = n^2(l-m).$$

447. Proposed by ELIJAH SWIFT, University of Vermont.

(*a*) A method is sought of forming an equation such that the first k figures of some root shall be given numbers. For example, form a cubic equation such that one of its roots is 1.918 +.

(*b*) Of all the equations suggested in (*a*), determine that one for which the sum of the absolute values of the coefficients is least.

GEOMETRY.

476. Proposed by CLIFFORD N. MILLS, Brookings, South Dakota.

Show that the locus of the middle points of a set of parallel chords intercepted between an hyperbola and its conjugate is $4b_1^2x^2y^2 - 4a_1^2y^4 = a_1^2b_1^4$.

Ashton's *Analytical Geometry*, p. 194, Prob. 34.

477. Proposed by N. P. PANDYA, Sojitra, India.

A village A_0 is equidistant from ten villages, $A_1, A_2, A_3, \cdots, A_{10}$. The distances, A_1A_2, A_2A_3, $\cdots$, A_9A_{10}, $A_{10}A_1$, are in arithmetic progression. A person, starting from A_0, has to go to four of these villages consecutively, and has then to return to A_0. What four villages should he select so that the total distance traveled by him may be a minimum?